Digital Communication Laboratory (Digital Comm Lab)

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Reference

- John G. Proakis, Masoud Salehi, and Gerhard Bauch. “Contemporary communication systems using MATLAB”. Third Edition, Cengage Learning, 2012.

Contents

Software

- MATLAB R2017b

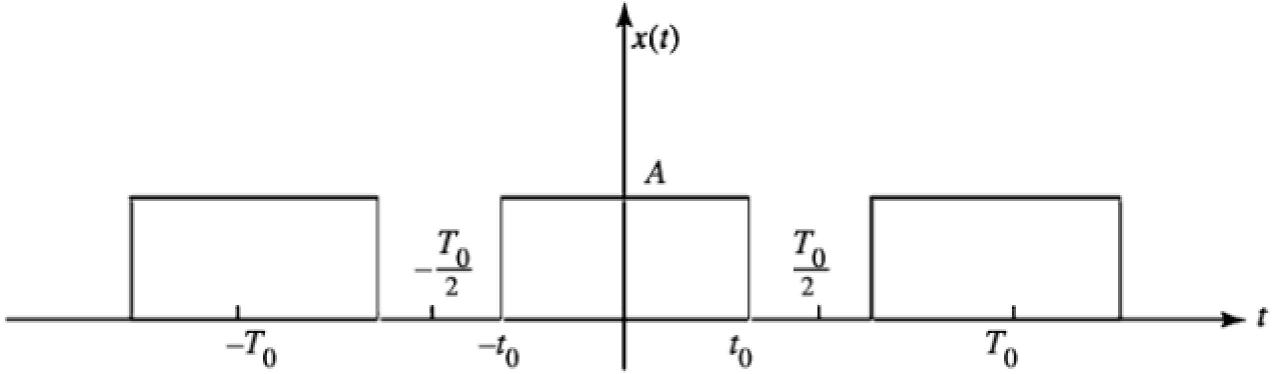
1. Signals and Linear Systems

- ILLUSTRATIVE PROBLEM 1.1

$$x(t) = A\Pi\left(\frac{t}{2t_0}\right) = \begin{cases} A, & |t| < t_0 \\ \frac{A}{2}, & t = \pm t_0 \\ 0, & \text{otherwise} \end{cases} \qquad (1.2.19)$$

for $|t| \le T_0/2$, where $t_0 < T_0/2$.

$$\Pi(t) = \begin{cases} 1, & |t| < \frac{1}{2} \\ \frac{1}{2}, & t = \pm\frac{1}{2} \\ 0, & \text{otherwise} \end{cases} \qquad (1.2.20)$$



4. Analog-to-Digital Conversion

- Huffman Coding:

ILLUSTRATIVE PROBLEM

Illustrative Problem 4.1 [Huffman Coding] Design a Huffman code for a source with alphabet $\mathscr{X} = \{x_1, x_2, \ldots, x_9\}$ and corresponding probability vector

$$\boldsymbol{p} = (0.2, 0.15, 0.13, 0.12, 0.1, 0.09, 0.08, 0.07, 0.06)$$

Find the average codeword length of the resulting code and compare it with the entropy of the source.

$$H(X) = -\sum_{x \in \mathscr{X}} p(x) \log p(x)$$

4. Analog-to-Digital Conversion

ILLUSTRATIVE PROBLEM

Illustrative Problem 4.2 [Huffman Coding] A discrete-memoryless information source with alphabet

$$\mathscr{X} = \{x_1, x_2, \ldots, x_6\}$$

and the corresponding probabilities

$$\boldsymbol{p} = \{0.1, 0.3, 0.05, 0.09, 0.21, 0.25\}$$

is to be encoded using Huffman coding.

1. Determine the entropy of the source.
2. Find a Huffman code for the source and determine the efficiency of the Huffman code.
3. Now design a Huffman code for source sequences of length 2 and compare the efficiency of this code with the efficiency of the code derived in part 2.

4. Analog-to-Digital Conversion

- 4.3. Quantization
  - Scalar Quantization
    - Uniform Quantization
    - Nonuniform Quantization
  - Vector Quantization

4. Analog-to-Digital Conversion

- 4.3. Quantization
  - Scalar Quantization
    - Uniform Quantization
    - Nonuniform Quantization: Many physical signals, such as speech signals, have the characteristic that small signal amplitudes occur more frequently than large signal amplitudes. However, a uniform quantizer provides the same spacing between successive levels throughout the entire dynamic range of the signal. A better approach would be to have a nonuniform quantizer, which provides more closely spaced levels at the small signal amplitudes and more widely spaced levels at the large signal amplitudes.

    - A nonuniform quantizer characteristic is usually obtained by passing the signal through a nonlinear device that compresses the signal amplitudes, followed by a uniform PCM quantizer.

    - Compressor + Expander = Compander

4. Analog-to-Digital Conversion

- 4.3. Quantization
  - Nonuniform Quantization:

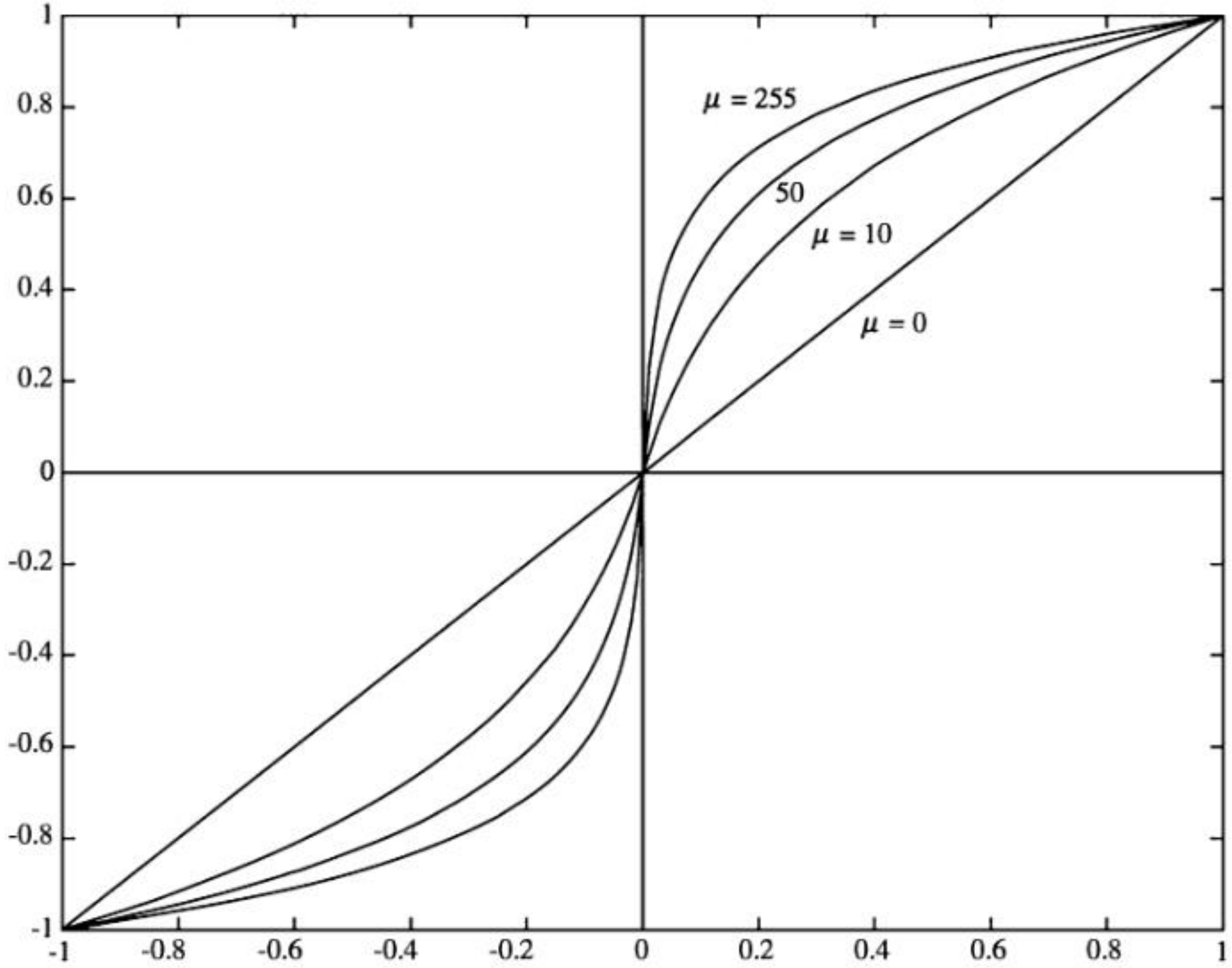


Figure 4.14: The μ-law compander

- MATLAB:
  - Source Coding
    - Compand a signal
      - Compand
        - 'mu/compressor'

4. Analog-to-Digital Conversion

- 4.3.3. Pulse Code Modulation
  - PCM
    - Sampling at a rate higher than Nyquist rate
    - Quantization
      - Uniform PCM
      - Nonuniform PCM

4. Analog-to-Digital Conversion

- 4.3.4. Differential Pulse Code Modulation (DPCM)
  - Samples are usually correlated random variables
  - In the simplest form of DPCM, difference between two adjacent samples is quantized.

$$Y_n = X_n - \hat{Y}'_{n-1}$$

$$\hat{Y}'_n = \hat{Y}_n + \hat{Y}'_{n-1}$$

$$\begin{aligned}\hat{Y}_n - Y_n &= \hat{Y}_n - (X_n - \hat{Y}'_{n-1}) \\ &= \hat{Y}_n - X_n + \hat{Y}'_{n-1} \\ &= \hat{Y}'_n - X_n\end{aligned}$$

$$\hat{X}_n = \hat{Y}_n + \hat{X}_{n-1}$$

$$\hat{Y}_n - Y_n = \hat{X}_n - X_n$$

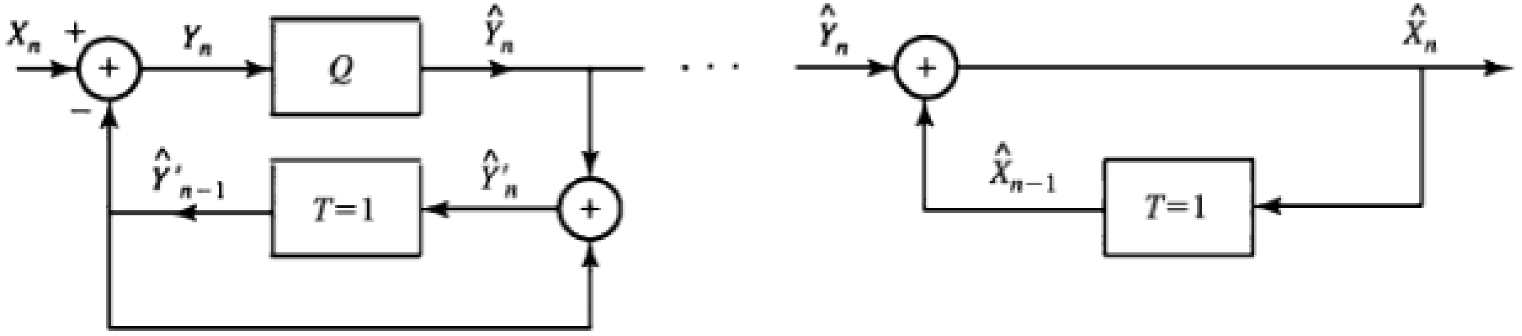


Figure 4.18: A simple DPCM encoder and decoder

4. Analog-to-Digital Conversion

- 4.3.4. Differential Pulse Code Modulation (DPCM)
  - MATLAB
    - Source Coding
      - DPCM

    - Predictor
    - dpcmenco
    - dpcmdeco
    - dpcmopt

5. Baseband Digital Transmission

- Binary Signal Transmission:

$$0 \to s_0(t), \qquad 0 \le t \le T_b$$
$$1 \to s_1(t), \qquad 0 \le t \le T_b$$

- Additive White Gaussian Noise (AWGN):

$$r(t) = s_i(t) + n(t), \qquad i = 0, 1, \qquad 0 \le t \le T_b$$

5. Baseband Digital Transmission

- Optimum Receiver for AWGN Channel:
  - Signal Correlator
  - Matched Filter

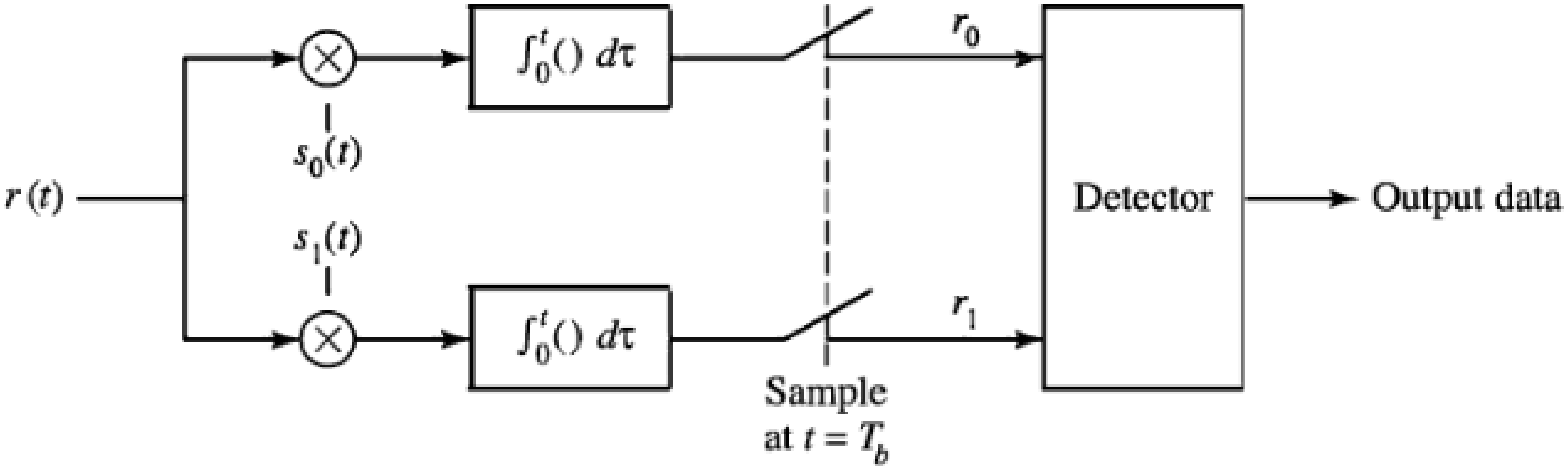
  - Detector

5. Baseband Digital Transmission

- Optimum Receiver for AWGN Channel:
  - Signal Correlator

$$r_0(t) = \int_0^t r(\tau) s_0(\tau)\, d\tau$$

$$r_1(t) = \int_0^t r(\tau) s_1(\tau)\, d\tau$$



5. Baseband Digital Transmission

- Optimum Receiver for AWGN Channel:
  - Signal Correlator
  - Example 1:

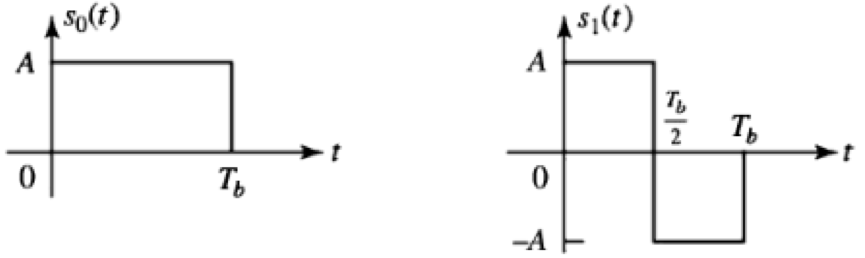


Figure 5.2: Signal waveforms $s_0(t)$ and $s_1(t)$ for a binary communication system

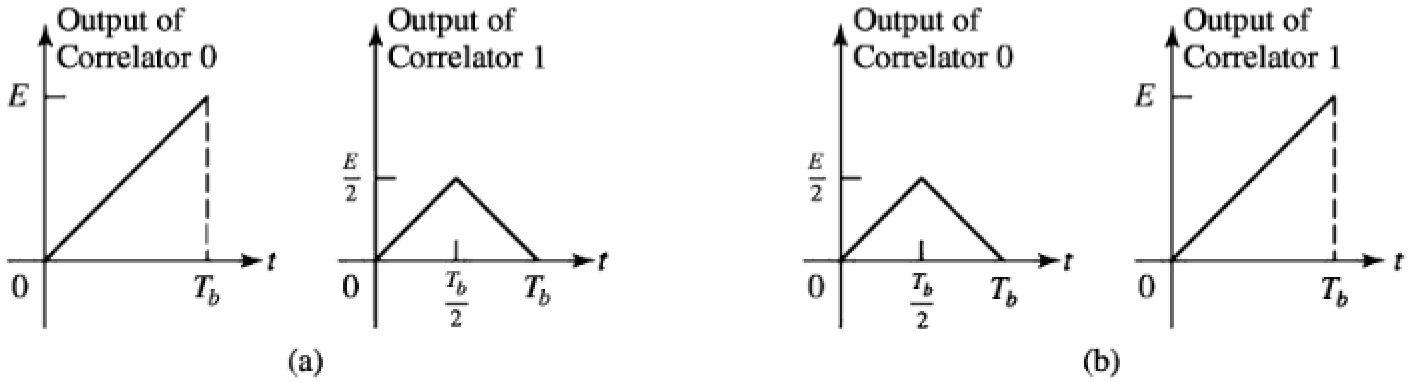


Figure 5.3: Noise-free correlator outputs. (a) $s_0(t)$ was transmitted. (b) $s_1(t)$ was transmitted

5. Baseband Digital Transmission

- Optimum Receiver for AWGN Channel:
  - Signal Correlator
  - Example 1:

$$
\begin{aligned}
r_0 &= \int_0^{T_b} r(t) s_0(t)\, dt \\
&= \int_0^{T_b} s_0^2(t)\, dt + \int_0^{T_b} n(t) s_0(t)\, dt \\
&= E + n_0
\end{aligned}
$$

$$
\begin{aligned}
r_1 &= \int_0^{T_b} r(t) s_1(t)\, dt \\
&= \int_0^{T_b} s_0(t) s_1(t)\, dt + \int_0^{T_b} n(t) s_1(t)\, dt \\
&= n_1
\end{aligned}
$$

$$
\begin{aligned}
n_0 &= \int_0^{T_b} n(t) s_0(t)\, dt \\
n_1 &= \int_0^{T_b} n(t) s_1(t)\, dt
\end{aligned}
$$

$$
\begin{aligned}
p(r_0 \mid s_0(t) \text{ was transmitted}) &= \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(r_0 - E)^2 / 2\sigma^2} \\
p(r_1 \mid s_0(t) \text{ was transmitted}) &= \frac{1}{\sqrt{2\pi}\,\sigma} e^{-r_1^2 / 2\sigma^2}
\end{aligned}
$$

5. Baseband Digital Transmission

- Optimum Receiver for AWGN Channel:
  - Signal Correlator
  - Example 1:

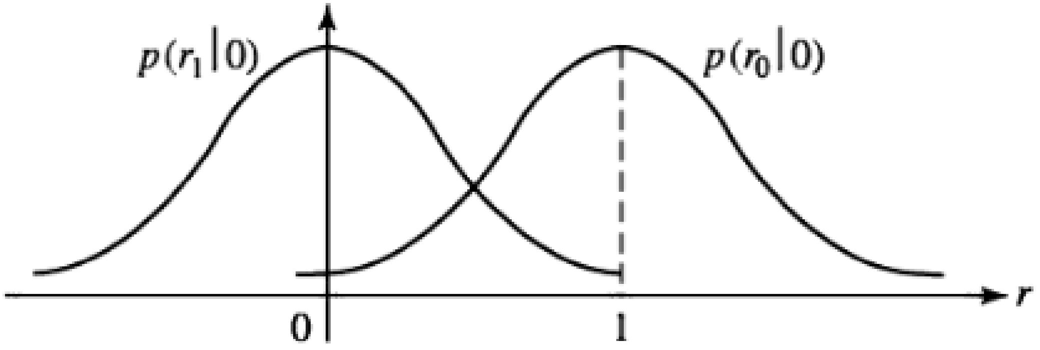


Figure 5.4: Probability density functions $p(r_0 \mid 0)$ and $p(r_1 \mid 0)$ when $s_0(t)$ is transmitted

5. Baseband Digital Transmission

- Optimum Receiver for AWGN Channel:
  - Signal Correlator
  - Example 2:

Illustrative Problem 5.2 [Correlation of Signal Waveforms] Sample the signal waveforms in Illustrative Problem 5.1 at a rate $F_s = 20/T_b$ (sampling interval $T_s = T_b/20$) and perform the correlation of $r(t)$ with $s_0(t)$ and $s_1(t)$ numerically; that is, compute and plot

$$r_0(kT_s) = \sum_{n=1}^{k} r(nT_s)s_0(nT_s), \quad k = 1, 2, \ldots, 20$$

and

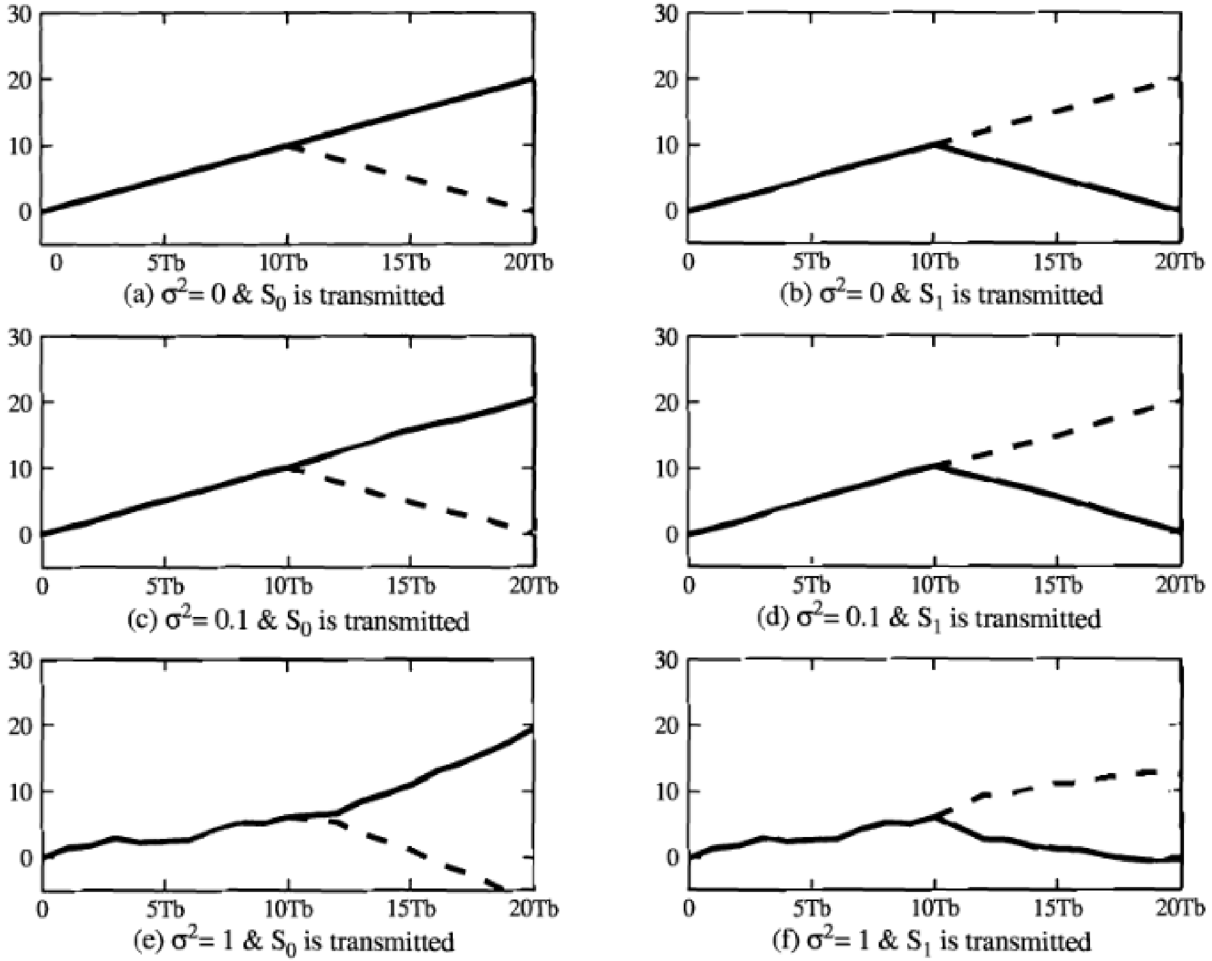
$$r_1(kT_s) = \sum_{n=1}^{k} r(nT_s)s_1(nT_s), \quad k = 1, 2, \ldots, 20$$

when (a) $s_0(t)$ is transmitted signal and (b) $s_1(t)$ is the transmitted signal.

Repeat the above computations and plots when the signal samples $r(kT_s)$ are corrupted by additive white Gaussian noise samples $n(kT_s)$, $1 \leq k \leq 20$, which have zero mean and variance $\sigma^2 = 0.1$ and $\sigma^2 = 1$.

5. Baseband Digital Transmission

- Optimum Receiver for AWGN Channel:
  - Signal Correlator
  - Example 2:



(a) $\sigma^2 = 0$ & S_0 is transmitted

(b) $\sigma^2 = 0$ & S_1 is transmitted

(c) $\sigma^2 = 0.1$ & S_0 is transmitted

(d) $\sigma^2 = 0.1$ & S_1 is transmitted

(e) $\sigma^2 = 1$ & S_0 is transmitted

(f) $\sigma^2 = 1$ & S_1 is transmitted

5. Baseband Digital Transmission

- Optimum Receiver for AWGN Channel:
  - Matched Filter

$$h(t) = s(T_b - t), \qquad 0 \le t \le T_b$$

$$y(t) = \int_0^t s(\tau)h(t-\tau)\,d\tau$$

$$y(t) = \int_0^t s(\tau)s(T_b - t + \tau)\,d\tau$$

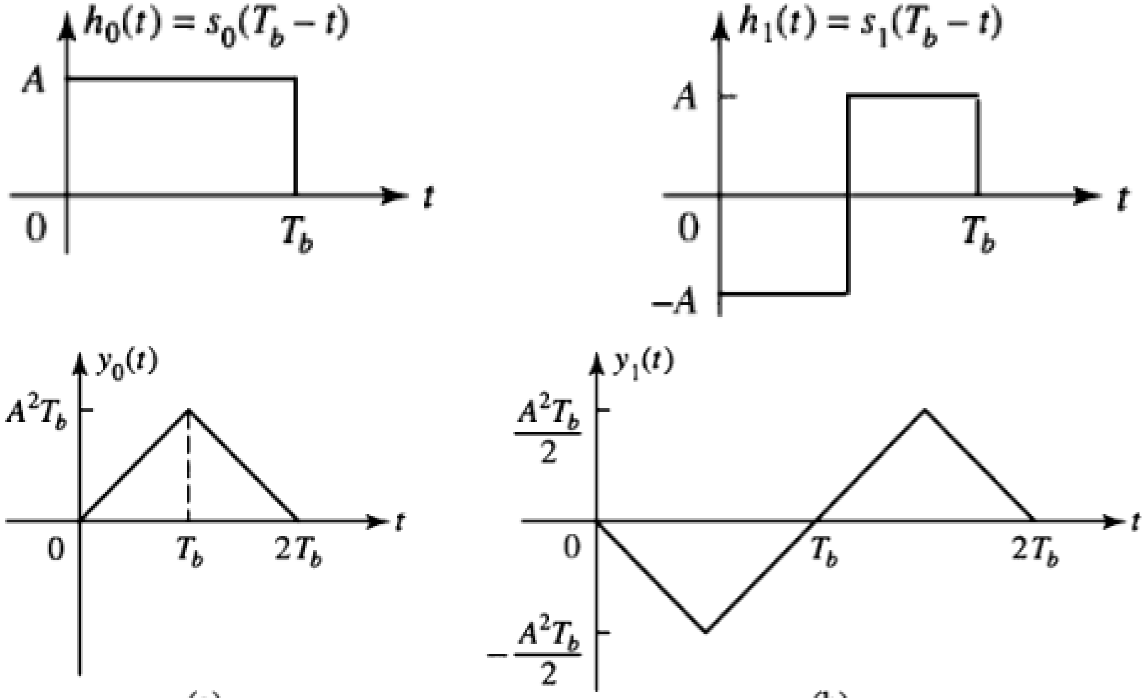
$$y(T_b) = \int_0^{T_b} s^2(t)\,dt = E$$

5. Baseband Digital Transmission

- Optimum Receiver for AWGN Channel:
  - Matched Filter
  - Example 1:

$$h_0(t) = s_0(T_b - t)$$

$$h_1(t) = s_1(T_b - t)$$



(a) (b)

5. Baseband Digital Transmission

- Optimum Receiver for AWGN Channel:
  - Matched Filter
  - Example 2:

Illustrative Problem 5.4 [Match Filtering of Signal Waveforms] Sample the signal waveform in Illustrative Problem 5.3 at a rate of $F_s = 20/T_b$ and perform the matched filtering of the received signal $r(t)$ with $s_0(t)$ and $s_1(t)$ numerically; that is, compute and plot

$$y_0(kT_s) = \sum_{n=1}^{k} r(nT_s)s_0(kT_s - nT_s), \quad k = 1, 2, \ldots, 20$$

and

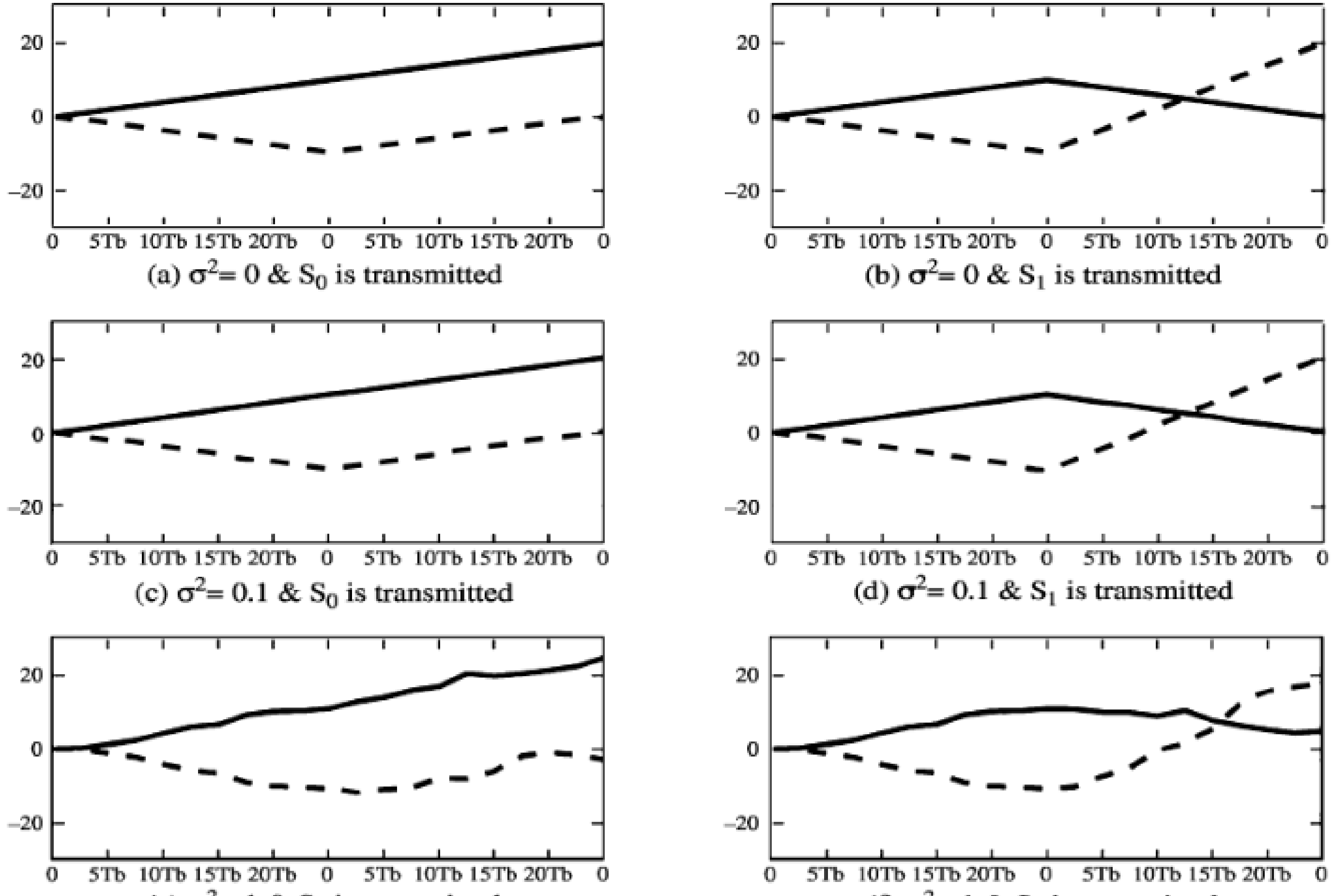
$$y_1(kT_s) = \sum_{n=1}^{k} r(nT_s)s_1(kT_s - nT_s), \quad k = 1, 2, \ldots, 20$$

when (a) $s_0(t)$ is the transmitted signal and (b) $s_1(t)$ is the transmitted signal.

Repeat the above computations when the signal samples $r(kT_s)$ are corrupted by additive white Gaussian noise samples $n(kT_s)$, $1 \le k \le 20$, which have zero mean and variance $\sigma^2 = 0.1$ and $\sigma^2 = 1$.

5. Baseband Digital Transmission

- Optimum Receiver for AWGN Channel:
  - Matched Filter
  - Example 2:



(a) $\sigma^2 = 0$ & S_0 is transmitted

(b) $\sigma^2 = 0$ & S_1 is transmitted

(c) $\sigma^2 = 0.1$ & S_0 is transmitted

(d) $\sigma^2 = 0.1$ & S_1 is transmitted

(e) $\sigma^2 = 1$ & S_0 is transmitted

(f) $\sigma^2 = 1$ & S_1 is transmitted

5. Baseband Digital Transmission

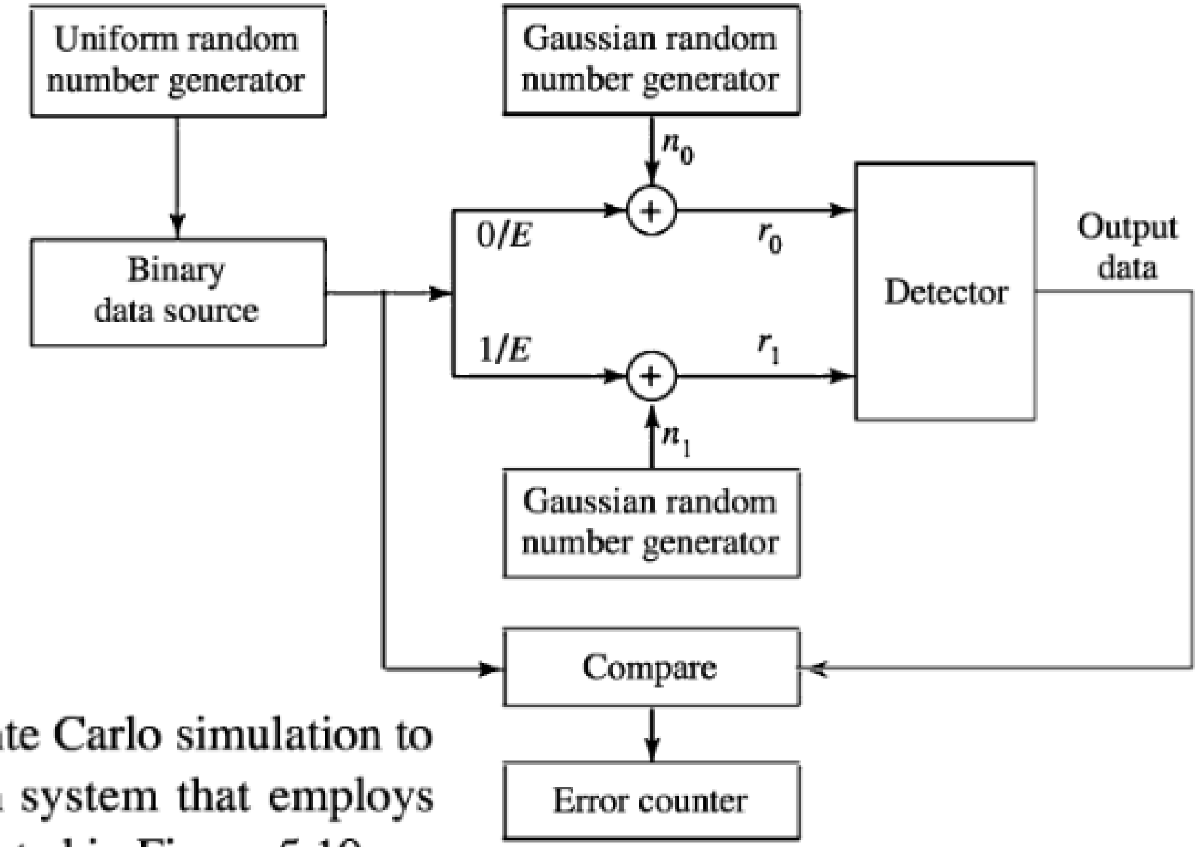
- Probability of error vs. signal-to-noise ratio

$$P_e = \frac{1}{\sqrt{2\pi}\,\sigma_x}\int_E^\infty e^{-x^2/2\sigma_x^2}\,dx$$
$$= \frac{1}{\sqrt{2\pi}}\int_{\sqrt{E/N_0}}^\infty e^{-x^2/2}\,dx$$
$$= Q\left(\sqrt{\frac{E}{N_0}}\right)$$

- Qfunc
- semilogy

5. Baseband Digital Transmission

- Monte Carlo Simulation of Binary Communication System



Illustrative Problem 5.6 [Monte Carlo Simulation] Use Monte Carlo simulation to estimate and plot P_e versus SNR for a binary communication system that employs correlators or matched filters. The model of the system is illustrated in Figure 5.10.

5. Baseband Digital Transmission

- Signal Constellation Diagrams for Binary Signals

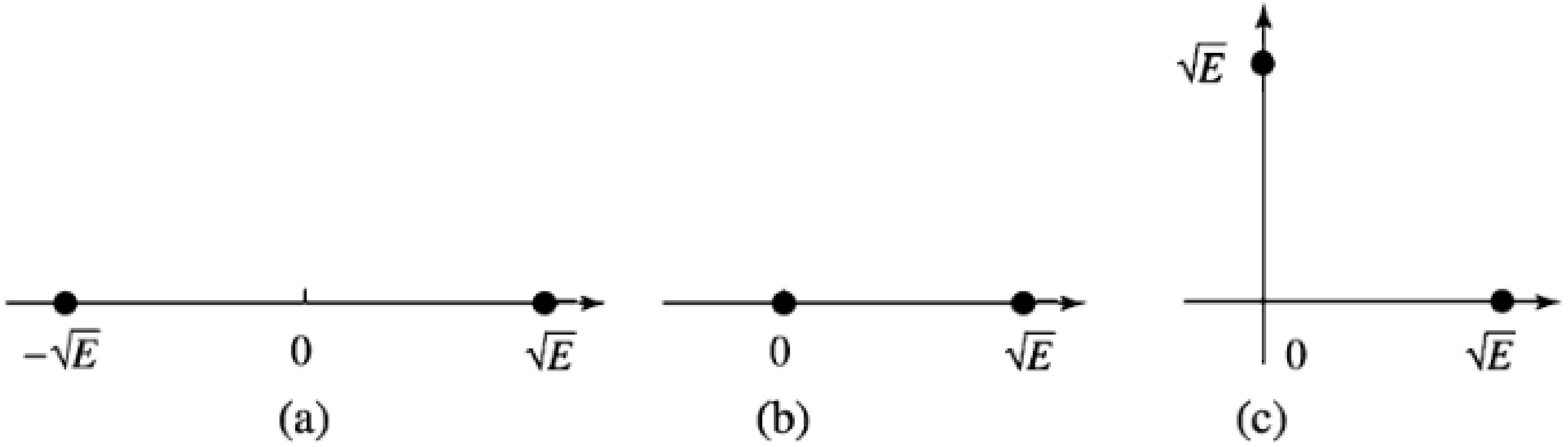


Figure 5.21: Signal constellations for binary signals. (a) Antipodal signals. (b) On–off signals. (c) Orthogonal signals

5. Baseband Digital Transmission

- Signal Constellation Diagrams for Binary Signals

Illustrative Problem 5.11 [Noise Effect on the Constellation] The effect of noise on the performance of a binary communication system can be observed from the received signal plus noise at the input to the detector. For example, let us consider binary orthogonal signals, for which the input to the detector consists of the pair of random variables (r_0, r_1), where either

$$(r_0, r_1) = (\sqrt{E} + n_0, n_1)$$

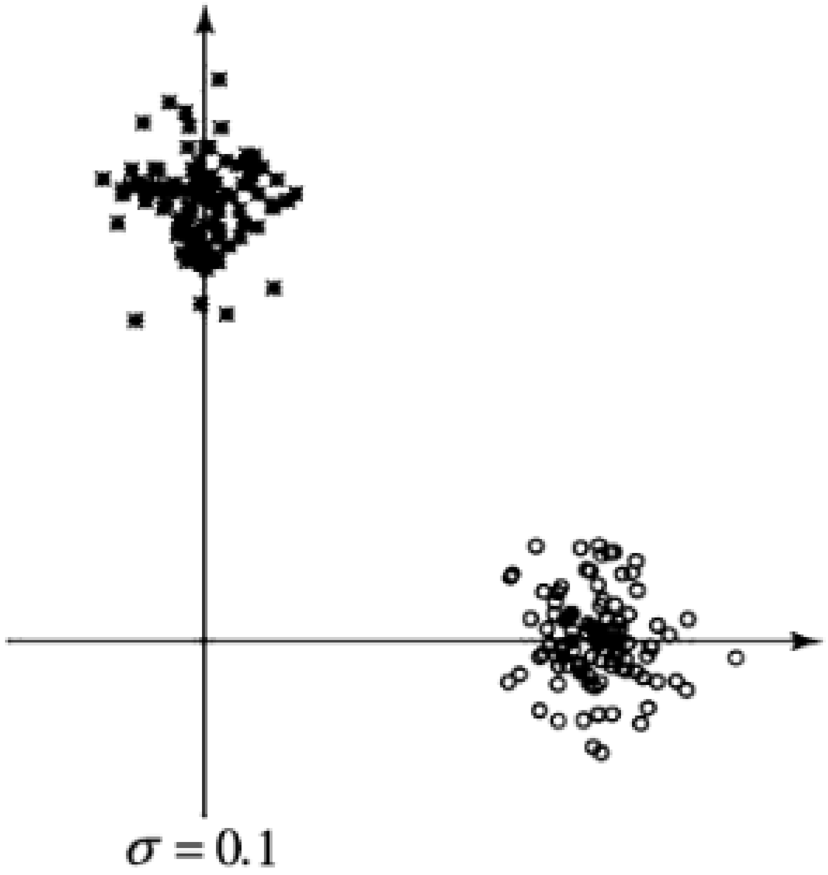
or

$$(r_0, r_1) = (n_0, \sqrt{E} + n_1)$$

The noise random variables n_0 and n_1 are zero-mean, independent Gaussian random variables with variance σ^2. As in Illustrative Problem 5.6, use Monte Carlo simulation to generate 100 samples of (r_0, r_1) for each value of $\sigma = 0.1$, $\sigma = 0.3$, and $\sigma = 0.5$, and plot these 100 samples for each σ on different two-dimensional plots. The energy E of the signal may be normalized to unity.

5. Baseband Digital Transmission

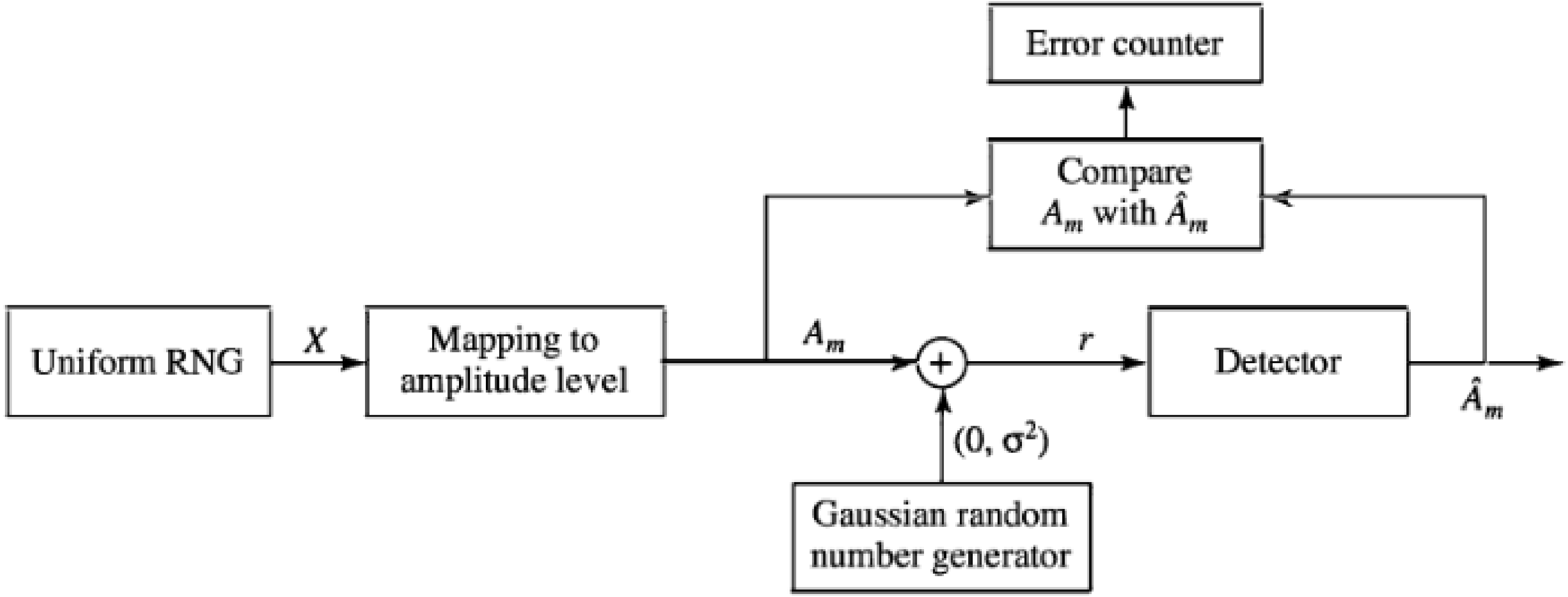
- Signal Constellation Diagrams for Binary Signals



5. Baseband Digital Transmission

- Multiamplitude Signal Simulation

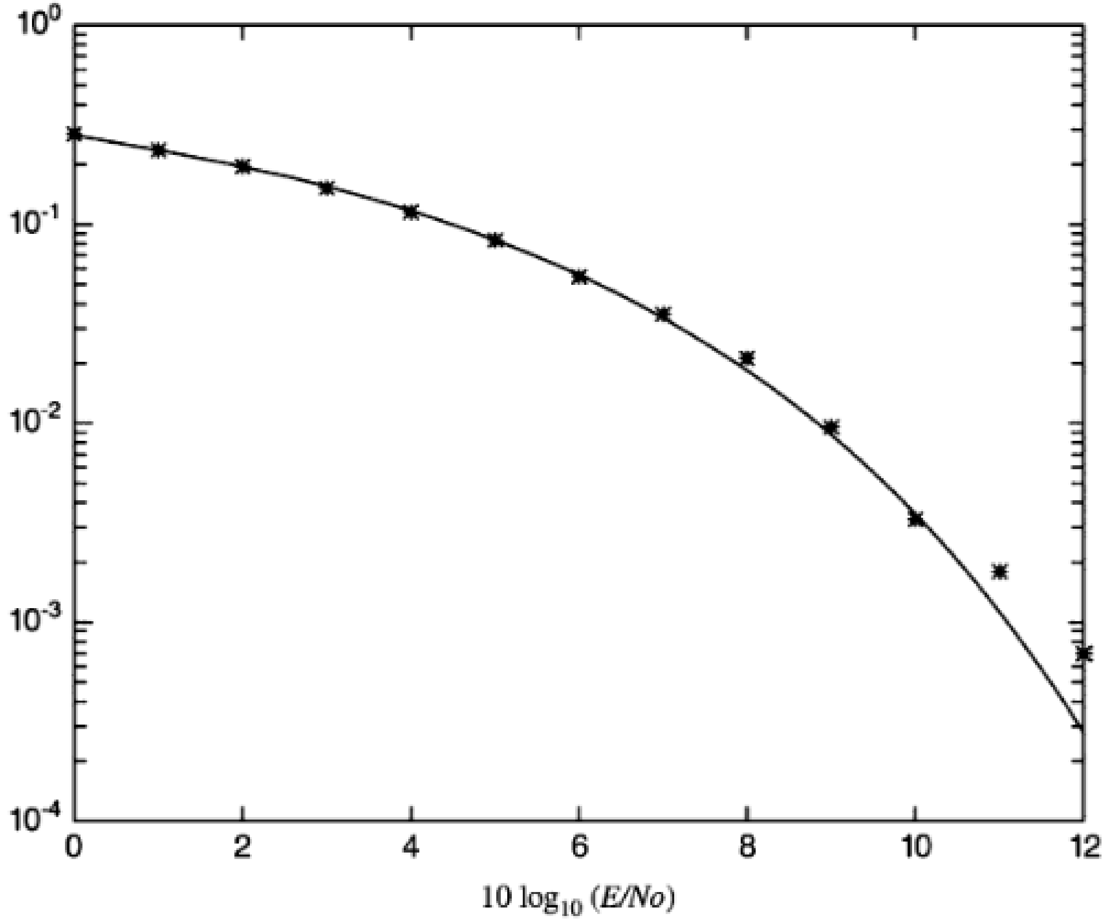
Illustrative Problem 5.12 [Multiamplitude Signal Simulation] Perform a Monte Carlo simulation of the four-level (quaternary) PAM communication system that employs a signal correlator, as described earlier, followed by an amplitude detector. The model for the system to be simulated is shown in Figure 5.26.



$$P_e = \frac{3}{2} Q\left(\sqrt{\frac{4}{5} SNR}\right)$$

5. Baseband Digital Transmission

- Multiamplitude Signal Simulation



5. Baseband Digital Transmission

- MATLAB Tool
  - bertool
  - doc_bpsk